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INFLUENCE OF PROFILE ANGLE ON FORCES DISTRIBUTION ON SILENT CHAIN TRANSMISSIONS

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***Abstract*—**The aim of this paper is to define and to analyze the theoretical contact forces that appear in a sprocket - silent chain joint. The contact force between the links and the teeth flanks of the sprocket can be considered as a reaction to the axial force (tensioning force) transmitted throughout the pins. The centrifugal force is considered to be constant, and the friction between the elements is not considered. This paper is presenting the steps to obtain the relationships between the forces from the sprocket-link contact depending on the tensioning force component between the links and the centrifugal force. These forces are computed to establish their distribution on links.

***Keywords*—** silent chain joint, sprocket, contact force, force distribution.

I. INTRODUCTION

CHAIN drive system's dynamics became of great interest to the automobile industry with the increase in the importance of noise, vibration, and harshness (NVH -noise, vibration, and harshness). Chain drives are used because of the non-slip, reliability and compact power transmission compared to belt transmissions, but usually with the disadvantages of increased noise and vibration. A brief history of chain drive systems and the important milestones in their practical use and development through the 20th century can be found in J. C. Conwell [1]. Timing chain transmissions are used more and more in combustion engines. Their robustness ensures the transmission of higher power, respectively higher torque for a longer maintenance-free period aggregate, S. Belmer, T. Fink, I. Lorenz and H. Neukirchner [2].

Silent chains as timing chains have a considerable interest regarding performance improvements of automobile engine systems, which can generate benefits like: reduced consumption of fuel and oil, higher engine power and torque, reduced harmful gas emissions, longer life and improved reliability of engines, K. Holmberg, P. Andersson and A. Erdemir [3], T. Hyakutake, M. Inagaki, M. Matsuda, N. Hakamada and Y. Teramachi [4], M. Schwaderlapp, F. Koch and J. Dohmen [5].

Silent chains are usually composed, as seen in Fig. 1. of 1 – inner plates, 2 – intermediate plates, 3 – outer plates and 4 – pin. The inner plates and the outer plates are fixed on the pin, while the intermediate plates are mounted with clearance on the pins. Only the inner and the intermediate plates are engaging with the teeth of sprockets. The outer plates are used for guiding. This type of chains is not standardized; there are different shapes and sizes of the plates; different number of inner and middle plates.

References of theoretical studies on silent chain transmissions come from early 20th century, J. R. Cautley [6]. A geometrical study of chain transmissions with two sprockets and a guide is presented in R. Papuc, R. Velicu and C. C. Gavrila [7]. A FEM study of forces distribution on links of a silent chain, at the contact with a circular guide, is presented in M. T. Lates, R. Papuc and C. C. Gavrila [8]; pressure distribution between a silent chain and a guide, resulted from a tribological study, is presented in R. Papuc, C. C. Gavrilă and M. Lates [9]. A comprehensive approach on general chain transmissions, including aspect of silent chains is in J. H. Pfalzgraf [10]; experimental results on load distribution between bush chains and sprockets is presented in M. R. Naji and K. Marshek [11]. This paper intends to study the theoretical contact which appears between the silent chain's elements and the sprockets.

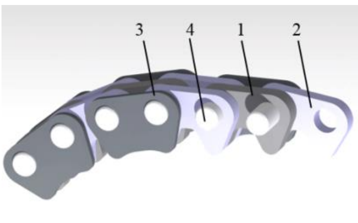

Fig. 1. Silent chain

II. THEORETICAL MODEL

Fig. 2. presents the theoretical model of the silent chain transmission. The middle or the inner plates of each link are coming in contact directly with the linear flank of the tooth, without rolling or sliding.

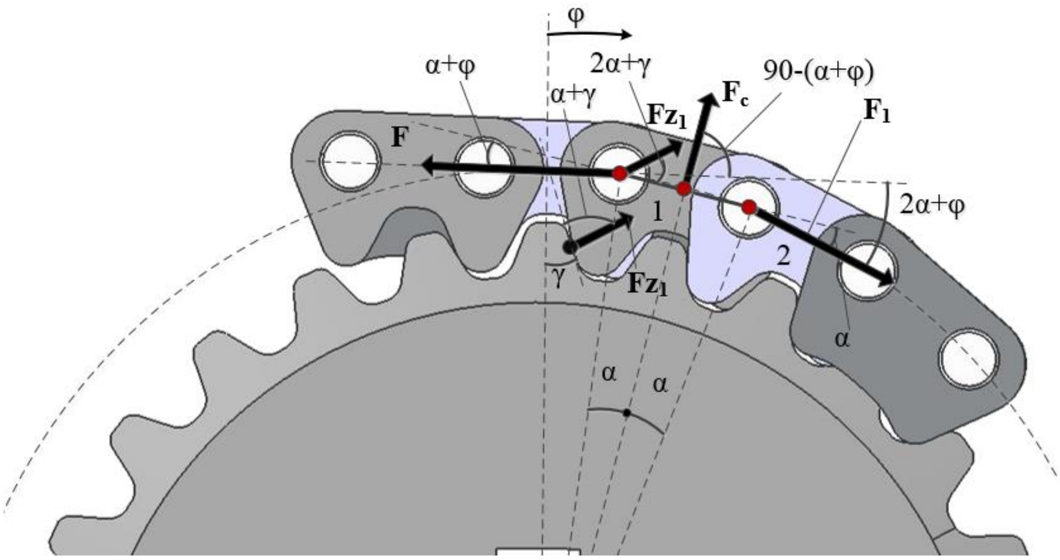

Fig. 2. Force distribution on silent chain transmission

F is the tensioning force applied to the first pin on the first link (1) of the chain. The theoretical force distribution takes into account the normal forces that appear between the sprocket tooth and chain plate of link 1 - F_{z1}, the tensioning force component between the silent chain links 1 and 2, that is transmitted by the second plate-pin joint – F_1, and the centrifugal force acting on link 1 that appear during the rotational speed of the chain transmission - F_c.

In this case, the force distribution is varying within an angle φ from -α to α, where α represents the half of the pitch angle on the sprocket. This angle depends on the number of teeth of the sprocket. The profile angle γ appears between the axis of a tooth and its flank line (plate's exterior flank). There will be taken into account three values for γ to find out what the effects of this angle are over the overall force transmission between the sprocket and the chain.

The theoretical model will be established using a constant rotational speed of the system, thus constant centrifugal force on the plates.

The calculus starts with the equilibrium of the forces written on two directions:

- on the direction of the tensioning force *F*:

$$\mathbf{F = F_{z1}\cos(\alpha+\gamma-\varphi) + F_c \sin(\varphi+\alpha) + F_1 \cos(\varphi+2\alpha)} \quad (1)$$

- on perpendicular direction over the tensioning force:

$$\mathbf{F_1 \sin(\varphi+2\alpha) = F_{z1}\sin(\alpha+\gamma-\varphi) + F_c \cos(\varphi+\alpha)} \quad (2)$$

Extracting from (2) the force from the plate-sprocket contact F_{z1} we have:

$$\mathbf{F_{z1} = \frac{F_1 \sin(\varphi+2\alpha) - F_c \cos(\varphi+\alpha)}{\sin(\alpha+\gamma-\varphi)}} . \quad (3)$$

Using (3) in (1) we get the equation for the component of the tensioning force between the chain links:

$$\mathbf{F_1 = \frac{F \sin(\alpha+\gamma-\varphi) + F_c \cos(2\alpha+\gamma)}{\sin(3\alpha+\gamma)}} . \quad (4)$$

Based on Fig. 2., the following equations can be drawn:

$$\mathbf{\alpha = \frac{360}{2z}} ; \quad (5)$$

$$\mathbf{v_t = \frac{\pi \cdot D_p \cdot n}{1000 \cdot 60}} ; \quad (6)$$

$$\mathbf{F_c = \frac{m_{chain} \cdot v_t^2}{\frac{D_p}{2} \cdot 1000}} . \quad (7)$$

Where *z* is the number of teeth on the sprocket, D_p is the pitch diameter of the sprocket, *n* is the rotational speed of the chain transmission, v_t is the tangential speed at the pitch diameter and m_{chain} is the mass of the silent chain.

For the equilibrium of link 2, the input force is the tensioning coming from link 1 F_1, which will replace the input F force used for the equilibrium of link 1. The relative angle between the links 1 and 2 being constant, angle φ in (3) and (4) will have the value of the pitch angle α.

The force F_1 for the second link will become F_2, while F_{z1} will become F_{z2}. All these equivalent replacements of the forces will be computed with the following observation: F_1 and F_{z1} are varying in function of the angle φ of link 1.

The tensioning force component for the second link can be described as:

$$\mathbf{F_2 = \frac{F_1 \sin(\gamma) + F_c \cos(2\alpha+\gamma)}{\sin(3\alpha+\gamma)}} . \quad (8)$$

The forces acting between the sprocket and the second link can be written as:

$$F_{z2} = \frac{F_2 \sin(3\alpha) - F_c \cos(2\alpha)}{\sin(\gamma)}. \tag{9}$$

For the next links, the forces can be computed with the following generalized formulas:

$$F_m = \frac{F_{m-1} \sin(\gamma) + F_c \cos(2\alpha + \gamma)}{\sin(3\alpha + \gamma)}; \tag{10}$$

$$F_{zm} = \frac{F_{m-1} \sin(3\alpha) - F_c \cos(2\alpha)}{\sin(2\gamma)}, \tag{11}$$

where *m* represents the number of the corresponding link of the silent chain.

III. RESULTS – FORCE DISTRIBUTION

The model for the force equilibrium presented above has been applied for $z = 27$ and three values of profile angle γ (20º, 25º, 30º). Table I presents the input parameters used to compute the theoretical forces that appear between the silent chain and the sprocket, calculated according to Schaeffler Group Automotive [12].

TABLE I

GEAR PARAMETERS $z = 27$, γ (20º, 25º, 30º).

Symbol	Unit	Value
z	-	27
n	rpm	5000
F	N	500
α	deg	6.66666
P	mm	8
D_{pitch}	mm	68.91032
M_{chain}	kg	0.0034
L_{chain}	mm	518
v_t	mm/s	9.51745
$F_{c/plate}$	N/plate	5.082994

The diagrams of tensioning force F_m depending on rotation angle φ, for three values of the contact angle γ, are presented for links 1, 2 an 3 in Fig. 3., Fig. 4. and Fig. 5. They are showing the continuous decrease with rotation angle and sequent number of links, with smaller values for smaller contact angle.The diagrams of contact forces F_{zm} depending on rotation angle φ, for three values of the contact angle γ, are presented for links 1, 2 and 3 in Fig. 6., Fig. 7. and Fig. 8. The contact force on the first contact increases with rotation angle and with a decrease of contact angle, but for the following contacts, there is a continuous decrease with rotation angle.

For the sequent number of links, the trend on the contact force is a faster decrease in smaller values of the contact angle. The reduction of contact angle determines a decrease of contact force starting with link 3. The second link is just a transition situation between the trend of the first link and the trend of the next links.

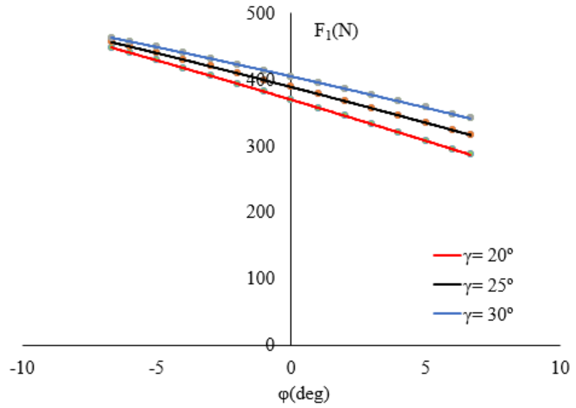

Fig. 3. Tensioning force distribution on the first link for three values of the profile angle γ (20º, 25º, 30º)

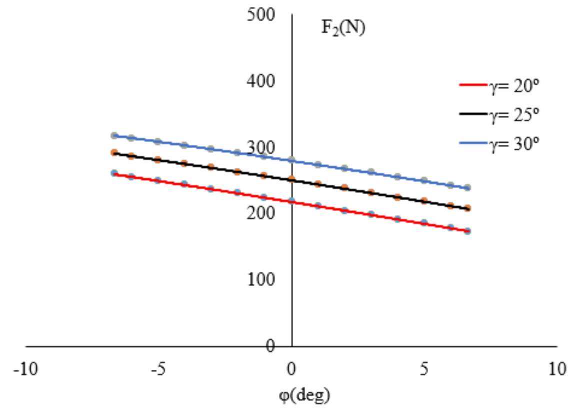

Fig. 4. Tensioning force distribution on the second link for three values of the profile angle γ (20º, 25º, 30º)

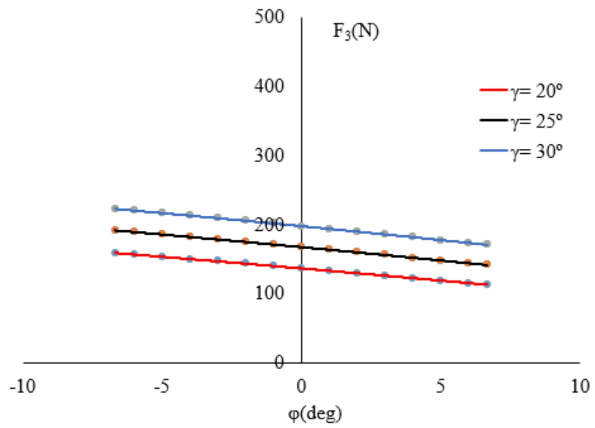

Fig. 5. Tensioning force distribution on the third link for three values of the profile angle γ (20º, 25º, 30º)

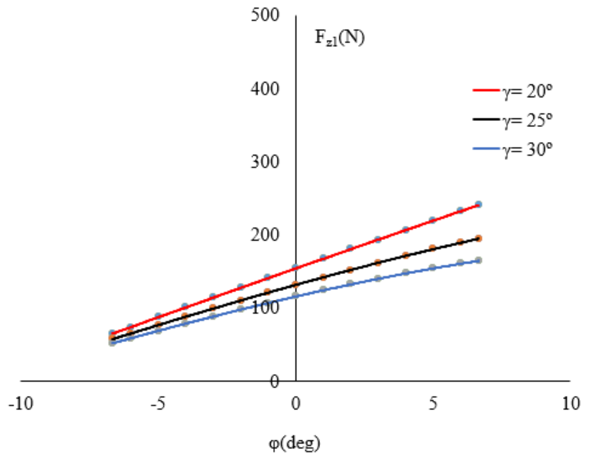

Fig. 6. The contact force on the first link for three values of the profile angle γ (20º, 25º, 30º)

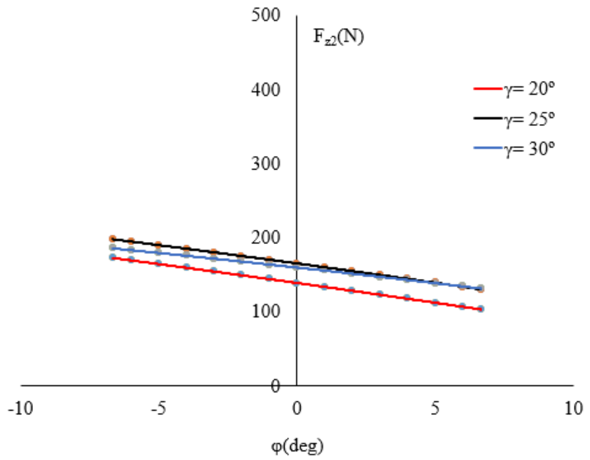

Fig. 7. The contact force distribution on the second link for three values of the profile angle γ (20º, 25º, 30º)

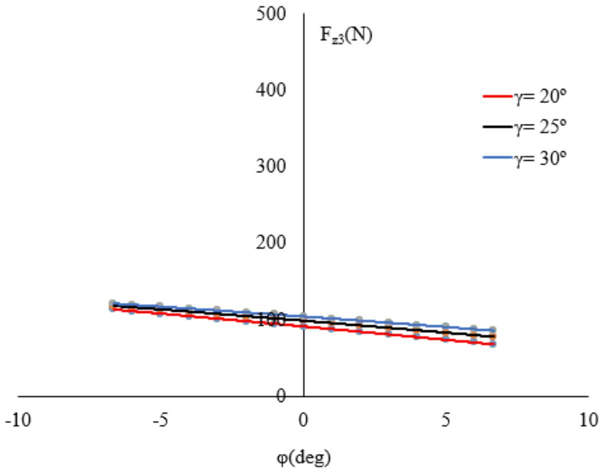

Fig. 8. The contact force distribution on the third link for three values of the profile angle γ (20º, 25º, 30º)

The analysis of the theoretical force distribution shows that the tensioning force in the silent chain transmission is distributed on more than the first couple of chain links.

The results of the theoretic analysis for the silent chain-sprocket show that:

1) When the forces get to negative values the chain gets off the sprocket – until that moment the force is transmitted between the chain links and the sprocket.

2) The profile angle of the silent chain plate (middle plate, inner plate) has a significant effect on the number contacts between the chain and the sprocket. As the profile angle increases the number of links in contact increases.

3) The F_z component from the chain transmission, as a reaction from the sprocket to the chain link, tends to move off the links from the sprocket.

IV. Conclusion

What the main finding from theoretical analyzes of the silent chain contact forces is that the number of chain links in contact is influenced by the profile angle of the tooth flank.

However, this conclusion must be proved when the friction forces are also considered.

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